

Investigations on CPT invariance at B -factories

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Abstract. We study the sensitivity of an asymmetric B -factory to indirect CPT violating effects. We find that both dilepton signals and nonleptonic asymmetries can be used to bound the space of parameters describing possible CPT violation in the mixing matrix of the neutral B system.

I Introduction

Invariance under CPT transforms is an exact property of a relativistic, local and renormalizable field theory, satisfying causality and invariance under proper Lorentz transformations, incorporating usual spin-statistic relations and provided with asymptotic states [1]. Therefore, tests of CPT invariance probe fundamental aspects of the current field-theoretical description of microscopic phenomena, and provide stringent constraints on quantum gravity, supergravity and superstring-inspired scenarios requiring a violation of CPT [2].

CPT symmetry, which implies the equality of mass, lifetime, branching ratios of particles and antiparticles, was tested at various levels of accuracy since the early days of the high-energy physics era [3]. The main activity was focused on the theoretical and experimental analysis of the neutral kaon system, where it was observed that the phases ϕ_{+-} , ϕ_{00} and ϕ_{SW} are equal within an accuracy of 1° [4]. At Φ -factories, where $8.9 \times 10^9 K_L - K_S$ pairs per year will be produced, tests of even higher accuracy are expected to be performed in the near future [5].

Tests of CPT invariance have been proposed for the neutral D meson system [6]. Moreover, the symmetric and asymmetric B -factories currently under construction [7–9] represent experimental facilities where CPT symmetry can be investigated in the neutral B system. In this case, information is used from $\Upsilon(4S)$ decays to coherent C -odd B^0 - \bar{B}^0 states (C -even states, e.g. from $\Upsilon(4S) \rightarrow B^0 \bar{B}^0 \gamma$, are expected at the level of $1/10^9$ events [10]) so that clear CPT tests, in addition to CP measurements, can be carried out.

The proposal of using the neutral B system for testing CPT was already put forward in [11, 12]. In particular, in [12] the features of symmetric and asymmetric B -factories were considered in order to put bounds to possible violations of CPT , mainly using the channel where one neutral B meson decays *via* a semileptonic transition, and the other one decays to $J/\psi K_S$; Monte Carlo simulations were used to study the sensitivity to this process in vari-

ous experimental environments. In this paper we want to reconsider the problem. In particular, we investigate the feasibility of bounding CPT invariance in the neutral B system at an asymmetric B -factory using not only nonleptonic decay channels, but also the decay modes where both the neutral B mesons decay via a semileptonic transition, since a high statistics is expected in this case, with limited reconstruction and background rejection difficulties. We introduce a CPT violating term in the mixing matrix of neutral B mesons, according to the notation in [11], and then we consider the case where lepton pairs are identified in semileptonic B^0 and \bar{B}^0 decays.

We also consider the decay channel $B \rightarrow J/\psi K_S$ and report on the sensitivity to different decay modes, such as $B \rightarrow D^+ D^-$ and $B \rightarrow D^{*+} D^{*-}$.

Our conclusion is that it is possible to sensibly constrain the parameter space of CPT violating effects in the neutral B mixing matrix. The future B -factories, therefore, represent powerful facilities for probing a fundamental aspect of the field theoretical description of the elementary interactions.

II Parameterizing CPT violation in the $B^0 - \bar{B}^0$ mixing matrix

In the Wigner-Weisskopf approach, the mixing B^0 - \bar{B}^0 is governed by the Hamiltonian

$$H = M - i\frac{\Gamma}{2} = \begin{pmatrix} M_{11} - i\frac{\Gamma_{11}}{2} & M_{12} - i\frac{\Gamma_{12}}{2} \\ M_{12}^* - i\frac{\Gamma_{12}^*}{2} & M_{22} - i\frac{\Gamma_{22}}{2} \end{pmatrix} \quad (2.1)$$

acting on the vector $\begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix}$. The condition $H_{11} \neq H_{22}$, or $\langle B^0 | H | B^0 \rangle \neq \langle \bar{B}^0 | H | \bar{B}^0 \rangle$, implies an indirect violation of CPT invariance in the neutral B system. Following [11] and [13], such a violation can be easily parameterized expressing the matrix (2.1) as follows:

$$H = -iD + \sigma_1 E_1 + \sigma_2 E_2 + \sigma_3 E_3 \quad (2.2)$$

with σ_i the Pauli matrices and D, E_1, E_2, E_3 four complex numbers. The correspondence between the parameters in (2.1-2.2) is simply given by:

$$\begin{aligned} E_1 &= \text{Re} M_{12} - \frac{i}{2} \text{Re} \Gamma_{12} \\ E_2 &= -\text{Im} M_{12} + \frac{i}{2} \text{Im} \Gamma_{12} \\ E_3 &= \frac{1}{2} (M_{11} - M_{22}) - \frac{i}{4} (\Gamma_{11} - \Gamma_{22}) . \end{aligned} \quad (2.3)$$

CP and CPT violating effects can be parameterized [11, 13] by introducing the new complex variables E, θ, ϕ , defined as:

$$\begin{aligned} E &= (E_1^2 + E_2^2 + E_3^2)^{1/2} , \quad E_1 = E \sin \theta \cos \phi , \\ E_2 &= E \sin \theta \sin \phi , \quad E_3 = E \cos \theta . \end{aligned} \quad (2.4)$$

As a matter of fact, CPT symmetry implies $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$, i.e. $\cos \theta = 0$. Moreover, using the phase convention $CP|B^0\rangle = |\bar{B}^0\rangle$, CP invariance requires the equality $\langle B^0|H|\bar{B}^0\rangle = \langle \bar{B}^0|H|B^0\rangle$, i.e.:

$$M_{12} - i \frac{\Gamma_{12}}{2} = M_{12}^* - i \frac{\Gamma_{12}^*}{2} . \quad (2.5)$$

This means $\text{Im} \Gamma_{12} = \text{Im} M_{12} = 0$, i.e. $\phi = 0$ and $\cos \theta = 0$.

The coefficients relating the mass eigenstates of the Hamiltonian (2.1) to the flavour eigenstates $|B^{(\pm)}\rangle$:

$$|B_1\rangle = \frac{1}{(|p_1|^2 + |q_1|^2)^{1/2}} (p_1|B^0\rangle + q_1|\bar{B}^0\rangle) \quad (2.6)$$

$$|B_2\rangle = \frac{1}{(|p_2|^2 + |q_2|^2)^{1/2}} (p_2|B^0\rangle - q_2|\bar{B}^0\rangle) \quad (2.7)$$

are simply related to the parameters θ and ϕ by the equations

$$\frac{q_1}{p_1} = e^{i\phi} \tan \frac{\theta}{2} , \quad \frac{q_2}{p_2} = e^{i\phi} \cot \frac{\theta}{2} . \quad (2.8)$$

Then, CPT invariance implies $\theta = \frac{\pi}{2}$ and

$$\frac{q_1}{p_1} = \frac{q_2}{p_2} = e^{i\phi} ; \quad (2.9)$$

a violation of CPT is governed by the parameter

$$s = \cot \theta = \frac{1}{2} \left(\frac{q_2}{p_2} - \frac{q_1}{p_1} \right) e^{-i\phi} . \quad (2.10)$$

Although, in principle, ϕ and θ are complex numbers, we assume in the following

$$\text{Im} \phi = 0 , \quad \text{Im} \theta = 0 , \quad (2.11)$$

and therefore a real variable s , to reduce the phenomenological analysis to the effects of the variation of a single parameter. We shall comment below on the possible effects related to the imaginary part of s .

If at $t = 0$ only a B^0 or \bar{B}^0 state is present, the time evolution of the mass eigenstates $|B_1\rangle$ and $|B_2\rangle$:

$$|B_n(t)\rangle = e^{-i(m_n - i\frac{\Gamma_n}{2})t} |B_n(0)\rangle \quad (n = 1, 2) \quad (2.12)$$

implies that

$$|B^0(t)\rangle = e^{-(im + \frac{\Gamma}{2})t} \left[g_+(t)|B^0\rangle + \bar{g}_+(t)|\bar{B}^0\rangle \right] \quad (2.13)$$

$$|\bar{B}^0(t)\rangle = e^{-(im + \frac{\Gamma}{2})t} [\bar{g}_-(t)|B^0\rangle + g_-(t)|\bar{B}^0\rangle] , \quad (2.14)$$

where [11]

$$g_{\pm}(t) = \cos^2 \frac{\theta}{2} e^{\pm(i\Delta m - \frac{\Delta\Gamma}{2})\frac{t}{2}} + \sin^2 \frac{\theta}{2} e^{\mp(i\Delta m - \frac{\Delta\Gamma}{2})\frac{t}{2}} \quad (2.15)$$

$$\begin{aligned} \bar{g}_{\pm}(t) &= \sin \frac{\theta}{2} \cos \frac{\theta}{2} \left[e^{(i\Delta m - \frac{\Delta\Gamma}{2})\frac{t}{2}} \right. \\ &\quad \left. - e^{-(i\Delta m - \frac{\Delta\Gamma}{2})\frac{t}{2}} \right] e^{\pm i\phi} ; \end{aligned} \quad (2.16)$$

$m_1(\Gamma_1)$ and $m_2(\Gamma_2)$ are the mass (width) of $|B_1\rangle$ and $|B_2\rangle$, respectively, and $m = \frac{m_1 + m_2}{2}$, $\Delta m = m_2 - m_1$, $\Gamma = \frac{\Gamma_1 + \Gamma_2}{2}$, $\Delta\Gamma = \Gamma_1 - \Gamma_2$.

Since $\frac{\Delta\Gamma}{\Gamma} \lesssim 10^{-2}$ [14], in the following we neglect the actual value of $\Delta\Gamma$.

Equations (2.13-2.16) show that the time evolution of neutral B states is governed also by the CPT violating parameter s , and then it is possible to define observables sensitive to s . A bound on s can be inferred from the mass difference $B^0 - \bar{B}^0$, using the relation

$$m_{B^0} - m_{\bar{B}^0} = \frac{s}{\sqrt{1 + s^2}} (m_1 - m_2) , \quad (2.17)$$

with $m_1 - m_2 = 3 \times 10^{-13} \text{ GeV}$ [3] the measured mass difference between B_1 and B_2 . A value $s \sim \mathcal{O}(10^{-1})$ implies a relative mass difference $\frac{|m_{B^0} - m_{\bar{B}^0}|}{m_{B \text{ average}}} \sim \mathcal{O}(10^{-15})$; in the neutral kaon system the bound $\frac{|m_{K^0} - m_{\bar{K}^0}|}{m_{K \text{ average}}} \leq 9 \times 10^{-19}$ [3] corresponds to $s \leq 2 \times 10^{-4}$.

III Dileptons

Let us now consider the signals of a possible CPT violation in various final states, starting from the processes where both the neutral B mesons decay *via* a semileptonic transition.

The $B^0\bar{B}^0$ wave function from $\Upsilon(4S)$ decays reads

$$\begin{aligned} |\psi(t)\rangle &= \frac{1}{\sqrt{2}} \left[|B^0(\mathbf{k}, t)\rangle + |\bar{B}^0(-\mathbf{k}, t)\rangle + C|B^0(-\mathbf{k}, t)\rangle \right. \\ &\quad \left. + |\bar{B}^0(\mathbf{k}, t)\rangle \right] \end{aligned} \quad (3.1)$$

where $\pm\mathbf{k}$ are the meson momenta in the $\Upsilon(4S)$ rest frame, and $C = -1$ is the charge conjugation of the $B^0\bar{B}^0$ pair.

Let us determine the probability of opposite sign dilepton production (l^+X^-), (l^-X^+), X^{\pm} being a generic hadronic state. If t_1 and t_2 are the decay times of the B

mesons with momentum \mathbf{k} and $-\mathbf{k}$, respectively, and if $P^\pm \mp(t_1, t_2)$ is the probability for a positive (negative) lepton at the time $t_1(t_2)$, one has:

$$P^{-+}(t_1, t_2) \propto | \langle l^- X^+, t_1; l^+ X^-, t_2 | \psi \rangle |^2 \quad (3.2)$$

$$\propto \frac{1}{2} \left| \langle l^- X^+, t_1 | H_W | B^0(t_1) \rangle \langle l^+ X^-, t_2 | H_W | \bar{B}^0(t_2) \rangle - \langle l^- X^+, t_1 | H_W | \bar{B}^0(t_1) \rangle \langle l^+ X^-, t_2 | H_W | B^0(t_2) \rangle \right|^2.$$

Since, within the Standard Model, only the transitions $\bar{B}^0 \rightarrow l^- X^+$, $B^0 \rightarrow l^+ X^-$ occur at the leading order in the Fermi constant, the following relations can be derived:

$$\langle l^+ X^-, t | H_W | \bar{B}^0(t) \rangle = e^{-(im+\frac{\Gamma}{2})\frac{t}{2}} \bar{g}_-(t) \ll \langle l^+ X^- | H_W | B^0 \rangle, \quad (3.3)$$

$$\langle l^- X^+, t | H_W | B^0(t) \rangle = e^{-(im+\frac{\Gamma}{2})\frac{t}{2}} \bar{g}_+(t) \ll \langle l^- X^+ | H_W | \bar{B}^0 \rangle, \quad (3.4)$$

$$\langle l^- X^+, t | H_W | \bar{B}^0(t) \rangle = e^{-(im+\frac{\Gamma}{2})\frac{t}{2}} g_-(t) \ll \langle l^- X^+ | H_W | \bar{B}^0 \rangle, \quad (3.5)$$

$$\langle l^+ X^-, t | H_W | B^0(t) \rangle = e^{-(im+\frac{\Gamma}{2})\frac{t}{2}} g_+(t) \ll \langle l^+ X^- | H_W | B^0 \rangle. \quad (3.6)$$

Neglecting violation of CPT in the decay amplitude, we can put:

$$\langle l^+ X^- | H_W | B^0 \rangle = \langle l^- X^+ | H_W | \bar{B}^0 \rangle = A_l. \quad (3.7)$$

Then, keeping only terms of $\mathcal{O}(s^2)$, the probability of production of opposite sign dileptons is given by:

$$P^{-+}(t_1, t_2) \propto \frac{1}{2} e^{-\Gamma(t_1+t_2)} (|A_l|^2)^2 |\bar{g}_+(t_1) \bar{g}_-(t_2) - g_-(t_1) g_+(t_2)|^2$$

$$= \frac{1}{2} (|A_l|^2)^2 \frac{e^{-\Gamma(t_1+t_2)}}{1+s^2} \times [1 + 2s^2 + \cos \Delta m(t_1 - t_2)]. \quad (3.8)$$

The production probability of equal sign dileptons can be derived in an analogous way; for $C = -1$ one has

$$P^{\pm\pm}(t_1, t_2) \propto (|A_l|^2)^2 \frac{e^{-\Gamma(t_1+t_2)}}{4(1+s^2)} [1 - \cos \Delta m(t_1 - t_2)]. \quad (3.9)$$

For $s = 0$, (3.8-3.9) reduce to well known expressions for the production probability of lepton pairs in B^0 - \bar{B}^0 decays. Time integration of (3.8) gives the integrated probability

$$P^{-+} \propto \frac{(|A_l|^2)^2}{\Gamma^2} \frac{1}{2(1+s^2)} \left[1 + 2s^2 + \frac{1}{1+x_d^2} \right], \quad (3.10)$$

where $x_d = \frac{\Delta m}{\Gamma}$. The quantity $\frac{(|A_l|^2)^2}{\Gamma^2}$ is proportional to the product of the semileptonic B^0 , \bar{B}^0 branching ratio;

the fractions $n^{\pm\pm}$ of equal and opposite sign dileptons produced for each decay can be easily derived:

$$n^{-+} = \frac{1}{2(1+s^2)} \left[1 + 2s^2 + \frac{1}{1+x_d^2} \right], \quad (3.11)$$

and

$$n^{++} = n^{--} = \frac{1}{4(1+s^2)} \frac{x_d^2}{1+x_d^2}. \quad (3.12)$$

The dependence on the parameter s signals CPT violation.

A measurement of the ratio

$$R = \frac{n^{++} + n^{--}}{n^{-+}} = \frac{x_d^2}{1 + (1 + 2s^2)(1 + x_d^2)} \quad (3.13)$$

provides a bound on s . Moreover, a measurement of time-dependent production, possible at an asymmetric B -factory, provides us with the fraction of opposite and equal sign dileptons:

$$n^{-+}(\Delta t) = \frac{e^{-\Gamma|\Delta t|}}{2(1+s^2)} [1 + 2s^2 + \cos \Delta m \Delta t] \quad (3.14)$$

$$n^{++}(\Delta t) = n^{--}(\Delta t) = \frac{e^{-\Gamma|\Delta t|}}{4(1+s^2)} [1 - \cos \Delta m \Delta t]. \quad (3.15)$$

Then, the ratio $R(\Delta t)$, analogous to (3.13), can be written as:

$$R(\Delta t) = \frac{n^{++}(\Delta t) + n^{--}(\Delta t)}{n^{-+}(\Delta t)} = \frac{1 - \cos \Delta m \Delta t}{1 + 2s^2 + \cos \Delta m \Delta t}. \quad (3.16)$$

It is worth observing that other asymmetries, obtained reconstructing the temporal sequences of production of equal and opposite-sign dileptons:

$$A = \frac{n^{++} - n^{--}}{n^{++} + n^{--}}, \quad \bar{A} = \frac{n^{-+} - n^{+-}}{n^{-+} + n^{+-}}, \quad (3.17)$$

vanish (both for positive and negative charge conjugation of the initial B^0 \bar{B}^0 states). However, this is only the case for a real s . In general, one finds [11]:

$$\bar{A}(\Delta t) = \frac{n^{-+}(\Delta t) - n^{+-}(\Delta t)}{n^{-+}(\Delta t) + n^{+-}(\Delta t)} = -\frac{2(Im s) \sin \Delta m |\Delta t|}{1 + \cos \Delta m \Delta t}, \quad (3.18)$$

and therefore from the asymmetry (3.18) a bound on $Im s$ can be derived.

It is now interesting to determine the upper bound on s that is possible to obtain in the BaBar experiment at the SLAC B -factory PEP II making use of dilepton events. At PEP II $N(B^0 \bar{B}^0) \simeq 1.8 \times 10^7$ $B^0 \bar{B}^0$ pairs are expected per year, for a machine running at the design luminosity [7]. Using the value of the semileptonic branching fraction $B(B^0 \rightarrow l^+ X^-) \simeq 0.105$, and the estimated lepton

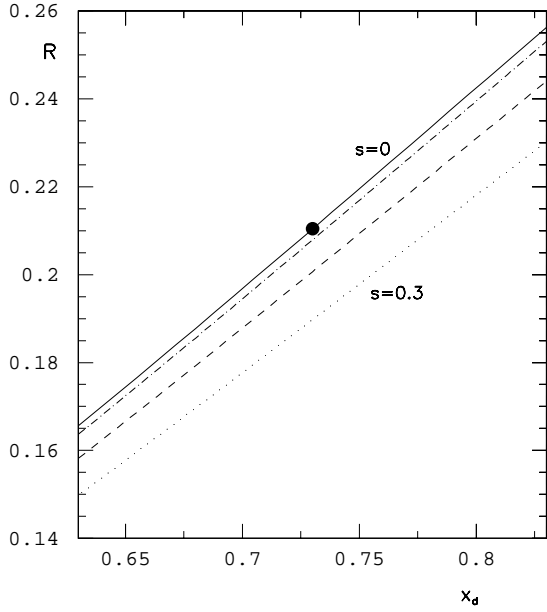


Fig. 1. The ratio R in (3.13) versus x_d and s . The *continuous line* corresponds to $s = 0$, the *dashed-dotted line* to $s = 0.1$, the *dashed line* to $s = 0.2$, the *dotted line* to $s = 0.3$

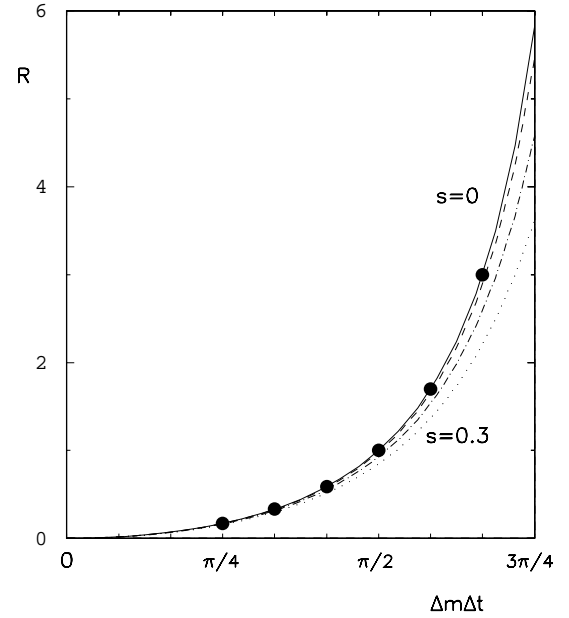


Fig. 2. Time dependent results for R (3.16). Notations as in Fig. 1. The expected statistical error is smaller than the size of the dots for the different values of $\Delta m \Delta t$

tagging efficiency $\epsilon_{tag} = 0.65$ [7], the number of expected dilepton events per year is:

$$N^{++} = N^{--} = N(B^0 \bar{B}^0) [B(B^0 \rightarrow l^+ X^-)]^2 \epsilon_{tag}^2 n^{++} \simeq 2.8 \times 10^4, \quad (3.19)$$

$$N^{-+} = N(B^0 \bar{B}^0) [B(B^0 \rightarrow l^+ X^-)]^2 \epsilon_{tag}^2 n^{-+} \simeq 2.8 \times 10^5. \quad (3.20)$$

The error on the variable R can be derived assuming a binomial distribution for $N^{\pm\pm}$ and N^{-+} . If p and $1-p$ are the probabilities of two leptons produced with opposite and equal charges:

$$p = n^{-+} = \frac{1}{2(1+s^2)} \left(1 + 2s^2 + \frac{1}{1+x_d^2} \right), \quad (3.21)$$

$$1-p = n^{++} + n^{--} = \frac{1}{2(1+s^2)} \frac{x_d^2}{1+x_d^2} \quad (3.22)$$

(for $s^2 \ll 1$), one has that the probability distribution

$$P(N^{-+}) = \binom{N}{N^{-+}} p^{N^{-+}} (1-p)^{N-N^{-+}}, \quad (3.23)$$

with $N = N^{++} + N^{--} + N^{-+}$, can be expressed in terms of R in the form

$$P\left(\frac{N}{1+R}\right) = \binom{N}{\frac{N}{1+R}} p^{\frac{N}{1+R}} (1-p)^{N-\frac{N}{1+R}}, \quad (3.24)$$

yielding

$$\sigma^2 \left(\frac{N}{1+R} \right) = Np(1-p) \quad (3.25)$$

and finally the error $\sigma(R)$:

$$\sigma(R) = (1+R)^2 \sqrt{\frac{2 N^{-+} N^{++}}{N^3}}. \quad (3.26)$$

In Fig. 1 the ratio R in (3.13) is plotted as a function of x_d and s in the ranges $0.63 < x_d < 0.83$ and $0 < s < 0.3$. In Fig. 2 we plot the time dependent ratio $R(\Delta t)$ in (3.14). The corresponding error bars have been calculated using (3.26). For time-integrated measurements, the error $\sigma(R)$ is given by $\sigma(R) = 9 \times 10^{-4}$ for $x_d = 0.73$, while, considering the whole range of x_d , $\sigma(R)$ runs from 8×10^{-4} to 1×10^{-3} .

These errors give an insight on the possible accuracy that can be obtained at a B -factory such as PEP II, due both to the machine luminosity and to the efficient lepton identification. The small value of $\sigma(R)$ shows that the region of possible values of x_d and s can be tightly constrained. The measurements of time integrated and time-dependent dilepton production fractions allow to establish an upper bound to the parameter s . As a matter of fact, from Fig. 2 we get that, for $\Delta m \Delta t = \frac{\pi}{4}$, the bounds $s < (5, 7, 8) \times 10^{-2}$ are obtained, respectively, within one, two and three standard deviations on $R(\Delta t)$; for $\Delta m \Delta t = \frac{\pi}{2}$ and $\frac{3\pi}{4}$ the bounds are $s < (4, 6, 7) \times 10^{-2}$, and $s < (4, 5, 6) \times 10^{-2}$, respectively.

IV Lepton-hadron channels

A Decay mode $B \rightarrow J/\psi K_S$

Now we wish to investigate the possibility of bounding CPT violating effects by means of nonleptonic decays of

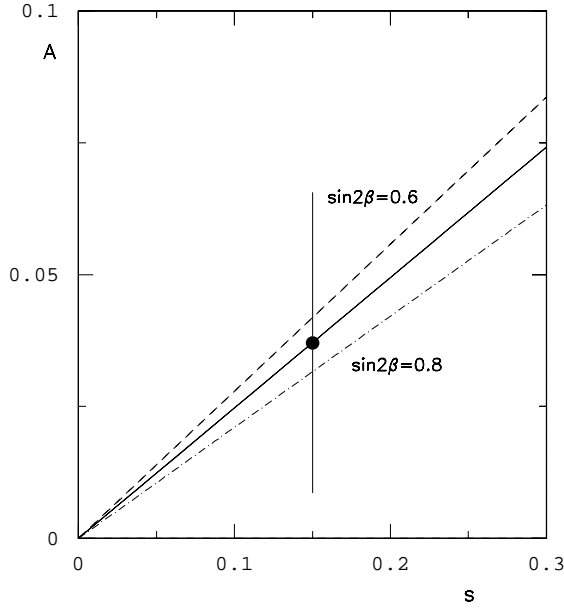


Fig. 3. Integrated asymmetry (4.5) for the channel $B^0 \rightarrow J/\psi K_S$. The three lines correspond to the values of the β angle: $\beta = 18^\circ$ (dashed-dotted line), $\beta = 22^\circ$ (solid line) and $\beta = 26^\circ$ (dashed line). Statistical errors show the minimum value of s accessible from this mode

neutral B mesons [12], using our parameterization of the CPT breaking term. We consider events where one B decays via a semileptonic transition to $l^\pm X^\mp$, while the other B decays into a nonleptonic state. On the experimental side, the semileptonic decay can be used to tag the flavour of the other B meson decaying into the hadronic final state $|f\rangle$.

Integrated and time-dependent CP asymmetries of the type

$$A = \frac{N(l^- X^+, f) - N(l^+ X^-, \bar{f})}{N(l^- X^+, f) + N(l^+ X^-, \bar{f})} \quad (4.1)$$

$$A(\Delta t) = \frac{N(l^- X^+, f; \Delta t) - N(l^+ X^-, \bar{f}; \Delta t)}{N(l^- X^+, f; \Delta t) + N(l^+ X^-, \bar{f}; \Delta t)} \quad (4.2)$$

can be used to test CPT invariance. Let us consider, for example, the hadronic decay $B^0 \rightarrow J/\psi K_S$. A tree and a penguin diagram contribute to such a process, so that we can express the transition amplitudes in the form:

$$\langle J/\psi K_S | H_W | B^0 \rangle = A_T e^{i\delta_T} e^{i\phi_T} + A_P e^{i\delta_P} e^{i\phi_P} \quad (4.3)$$

$$\langle J/\psi K_S | H_W | \bar{B}^0 \rangle = A_T e^{i\delta_T} e^{-i\phi_T} + A_P e^{i\delta_P} e^{-i\phi_P}, \quad (4.4)$$

where $A_{T,P}$ and $\phi_{T,P}$ are the weak amplitudes and phases respectively, while $\delta_{T,P}$ are the strong phases. At the leading order in s the CP asymmetries (4.1) and (4.2) read respectively

$$A = \frac{x_d^2}{1 + x_d^2} s \cos 2\beta, \quad (4.5)$$

$$A(\Delta t) = -\sin 2\beta \sin \Delta m \Delta t + s \cos 2\beta (1 - \cos \Delta m \Delta t), \quad (4.6)$$

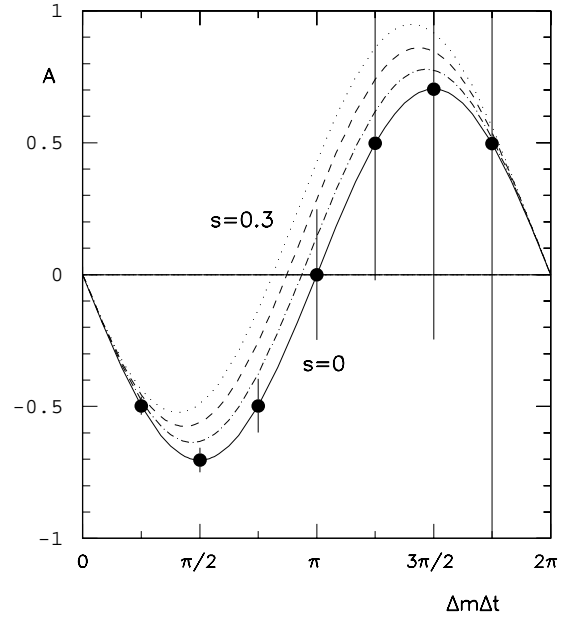


Fig. 4. Time dependent asymmetry (4.6) for the channel $B^0 \rightarrow J/\psi K_S$. The lines correspond to the values $s = 0$ (solid line), $s = 0.1$ (dashed-dotted line), $s = 0.2$ (dashed line) and $s = 0.3$ (dotted line). The expected statistical error for different values of $\Delta m \Delta t$ is also shown

in terms of the β angle of the unitarity triangle.

The sensitivity of PEP II to a CPT violation effect in this channel can be obtained noticing that the error on A is given by

$$\sigma(A) = d \sqrt{\frac{(1+A)(1-A)}{N_{eff}}}. \quad (4.7)$$

In (4.7) the effective number of events N_{eff} is given by: $N_{eff} = N(f)K(f, \bar{f})E_{tag}$, with $N(f)$ the number of events with a reconstructed hadronic state, $K(f, \bar{f}) =$

$n(f, l^+ X^-) + n(\bar{f}, l^- X^+)$, $n^{(-)}(f, l^\pm X^\mp)$ being the fractions of lepton-hadron events produced in the decay of the $B^0 \bar{B}^0$ pair, E_{tag} the total efficiency of tagging, d the dilution factor for the production of $|f\rangle$ states from events different

from $B^0 \rightarrow f$ decays. For the final state $f = J/\psi K_S$ the values $E_{tag} = 0.34$, $N(f) = 886$ and $d = 1$ are expected [7]. In Fig. 3 and Fig. 4 we plot integrated asymmetries for $\beta = 18^\circ$, $\beta = 22^\circ$ and $\beta = 26^\circ$, and time-dependent asymmetries for $\beta = 22^\circ$, with the corresponding error bars. We notice that the maximum value of the time-dependent asymmetry grows with s , and therefore higher order terms in the s -expansion are needed to fulfil the bound $|A| \leq 1$.

For integrated measurements at PEP II, the expected error on A is: $\sigma = 5.8 \times 10^{-2}$, quite independent of β . As a result, it is possible to bound $s \leq 0.21$ for $\beta = 18^\circ$, $s \leq 0.24$ for $\beta = 22^\circ$ and $s \leq 0.28$ for $\beta = 26^\circ$ within one standard deviation.

From time-dependent asymmetries, in the PEP II experimental environment the bounds $s \leq 0.13$ (0.26) (with-

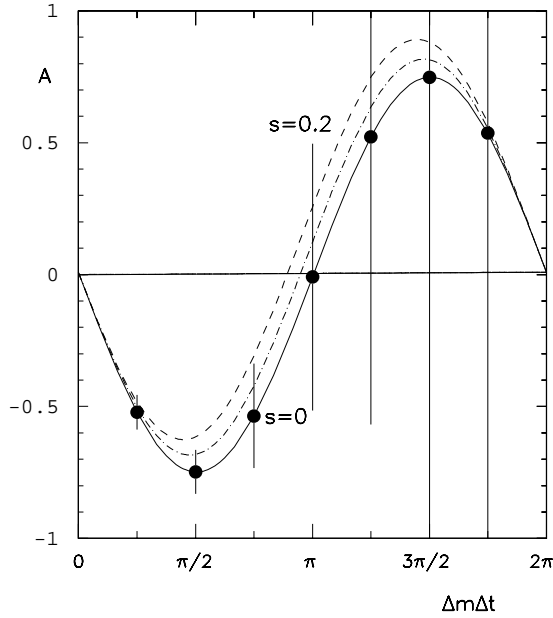


Fig. 5. Time dependent asymmetry (4.2) for the channel $B^0 \rightarrow D^+D^-$. The lines correspond to the values $s = 0$ (solid line), $s = 0.1$ (dashed-dotted line) and $s = 0.2$ (dashed line)

in one and two standard deviations) can be established in correspondence to $\Delta m \Delta t = \frac{\pi}{2}$, and $s \leq 0.17$ for $\Delta m \Delta t = \frac{3\pi}{4}$.

B Other decay modes

Other hadronic decay modes, such as $B \rightarrow D^+D^-$ and $B \rightarrow D^{*+}D^{*-}$, can be used to bound CPT invariance. To calculate the asymmetries for such final states, new information has to be considered concerning the transition amplitude and the final state interaction.

The transition amplitudes $B \rightarrow D^{(*)}D^{(*)}$ are governed by the effective hamiltonian [15]

$$H_{eff}(\Delta B = -1) = \frac{G}{\sqrt{2}} \left[V_{ub}V_{uq}^*(c_1O_1^u + c_2O_2^u) + V_{cb}V_{cq}^*(c_1O_1^c + c_2O_2^c) - V_{tb}V_{tq}^* \sum_{i=3}^6 c_i O_i \right], \quad (4.8)$$

where V_{jb} and V_{uq}^* ($j = u, c, t$; $q = d, s$) are elements of the Cabibbo-Kobayashi-Maskawa matrix, and c_i ($i = 1, \dots, 6$) the Wilson coefficients at the scale $\mu \simeq m_b$; $O_1^{u,c}$ and $O_2^{u,c}$ are current-current operators, O_i ($i = 3, \dots, 6$) are penguin operators.

In the vacuum saturation approximation, the decay amplitudes can be written as

$$\langle D^+D^- | H_W | B^0 \rangle = \frac{G}{\sqrt{2}} \left[V_{cd}V_{cb}^* a_1 - V_{td}V_{tb}^* \right]$$

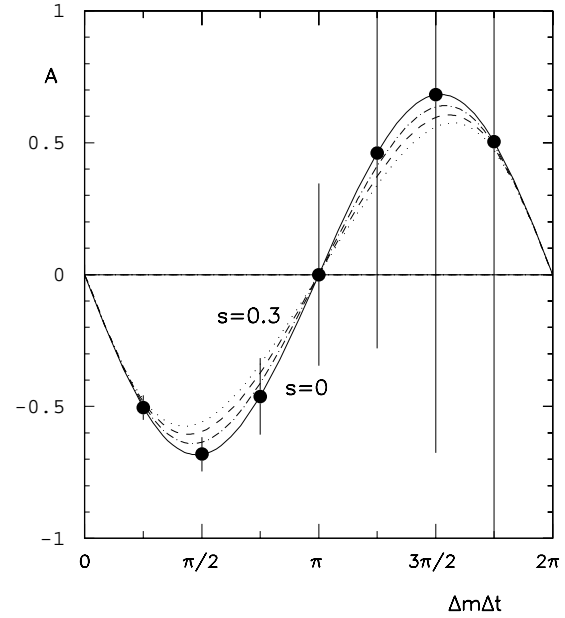


Fig. 6. Time dependent asymmetry (4.2) for the channel $B^0 \rightarrow D^{*+}D^{*-}$. The lines correspond to the values $s = 0$ (solid line), $s = 0.1$ (dashed-dotted line), $s = 0.2$ (dashed line) and $s = 0.3$ (dotted line)

$$\begin{aligned} & \times \left(a_4 + \frac{2a_6 m_D^2}{(m_b - m_c)(m_c + m_d)} \right) \\ & \times \langle D^+ | \bar{c} \gamma_\mu (1 - \gamma_5) d | 0 \rangle \\ & \times \langle D^- | \bar{b} \gamma^\mu (1 - \gamma_5) c | B^0 \rangle, \quad (4.9) \\ & \langle D^{*+} D^{*-} | H_W | B^0 \rangle = \frac{G}{\sqrt{2}} (V_{cd} V_{cb}^* a_1 - V_{td} V_{tb}^* a_4) \\ & \times \langle D^{*+} | \bar{c} \gamma_\mu (1 - \gamma_5) d | 0 \rangle \\ & \times \langle D^{*-} | \bar{b} \gamma^\mu (1 - \gamma_5) c | B^0 \rangle, \quad (4.10) \end{aligned}$$

with parameters a_1, \dots, a_6 related to the Wilson coefficients c_1, \dots, c_6 by the equations:

$$a_{2i-1} = c_{2i-1} + \frac{c_{2i}}{N_c}, \quad a_{2i} = c_{2i} + \frac{c_{2i-1}}{N_c} \quad (4.11)$$

($i = 1, 2, 3$ and N_c is the number of colours).

In the NDR renormalization scheme at the next-to-leading order, using $N_c = 3$ and the scale $\mu = m_b$, we get [15]

$$\begin{aligned} a_1 &= 1.017, \quad a_2 = 0.175, \quad a_3 = 0.0013, \\ a_4 &= -0.030, \quad a_5 = -0.0037, \quad a_6 = -0.038. \quad (4.12) \end{aligned}$$

Integrated and time-dependent asymmetries depend on the parameters of the Cabibbo-Kobayashi-Maskawa matrix and, unlike the $B^0 \rightarrow J/\psi K_S$ mode, also on strong final state phases. Various estimates, however, suggest that for B transitions to charmed states such strong phases are small [16], and therefore we neglect them in the analysis. The asymmetry (4.2) can be computed in a straightforward way; we omit here the lengthy expression, and only

plot in Fig. 5 and Fig. 6 the time dependent asymmetries for $B \rightarrow D^+D^-$ and $B \rightarrow D^{*+}D^{*-}$ respectively, for $\beta = 22^\circ$ and the parameters of the CKM matrix $\rho = 0.05$, $\eta = 0.39$, $\gamma = 83^\circ$; the corresponding error bars are also depicted in the figures.

The analysis shows that a sensible bound on s can be obtained from these decay channels only if tagging and reconstruction efficiencies at PEP II improve with respect to the estimates quoted in [7]. For instance, an improvement by a factor of two will allow to put the bounds $s \leq 0.18$ (from $B \rightarrow DD$) and $s \leq 0.22$ (from $B \rightarrow D^*D^*$) in correspondence to $\Delta m \Delta t = \frac{\pi}{2}$.

As for the hadronic channels $B^0 \rightarrow \pi^+\pi^-$ and $B^0 \rightarrow \rho^\pm\pi^\mp$, the CP asymmetries depend on the strong phases due to the hadron rescattering in the final state. In any case, we found that the statistics expected for such decay modes does not allow to sensitively bound the CPT violating parameter s .

V Testing direct CPT violation effects

So far we analyzed indirect CPT violation effects. It is also possible to consider observables which are sensitive to direct violation effects. One of them is the asymmetry between the partial widths of charged B mesons, related to a CPT violating term in the transition amplitudes $\langle f|H_W|B^+\rangle \neq \langle \bar{f}|H_W|B^-\rangle$:

$$A = \frac{\Gamma(B^+ \rightarrow f) - \Gamma(B^- \rightarrow \bar{f})}{\Gamma(B^+ \rightarrow f) + \Gamma(B^- \rightarrow \bar{f})}. \quad (5.1)$$

We do not parameterize this kind of violation, but just compute the expected error on the asymmetry (5.1). For the decay mode $B^+ \rightarrow \bar{D}^0 l^+ \nu_l$, $N = 2.9 \times 10^5$ identified events are expected per year [7], so that the estimated error on (5.1) is $\sigma = 7.9 \times 10^{-3}$; for $B^+ \rightarrow \bar{D}^{0*} l^+ \nu_l$, the number of expected events per year is $N = 3.9 \times 10^6$, and the estimated error is $\sigma = 5.1 \times 10^{-3}$.

VI Conclusions

We investigated the possibility of testing CPT symmetry in the neutral B meson system, considering the experimental environment provided by the B -factory PEP II. We found that dileptons and lepton-hadron events, with the hadronic decay $B^0 \rightarrow J/\psi K_S$, can be used to constrain the CPT violating parameter s .

In particular, from equal and opposite sign dilepton production, one can derive the bound $s \leq \mathcal{O}(10^{-2})$, which corresponds to a relative mass difference $\frac{|m_{B^0} - m_{\bar{B}^0}|}{m_{B_{average}}} \leq \mathcal{O}(10^{-16})$.

As for the hadronic decay $B^0 \rightarrow J/\psi K_S$, the sensitivity of PEP II is up to $s \simeq \mathcal{O}(10^{-1})$. Our analysis, though being quite different, confirms the estimate in [12] about the order of CPT violating effects that can be constrained at PEP II.

Other channels, such as $B^0 \rightarrow D^+D^-$ and $B^0 \rightarrow D^{*+}D^{*-}$, might allow a sensible bounding of s if tagging and reconstruction efficiencies improve at least by a factor of two with respect to the current expectations. Charmless decays of the type $B^0 \rightarrow \pi^+\pi^-$, $B^0 \rightarrow \rho^\pm\pi^\mp$, turn out to be unsuitable for testing CPT .

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Appendix A: Case $C = +1$

For the sake of completeness, we report the relevant formulae corresponding to a C even B^0 - \bar{B}^0 state:

$$P_{+}^{-+} \propto \frac{1}{4}(|A_l|^2)^2 \frac{e^{-\Gamma(t_1+t_2)}}{4(1+s^2)^2} \{1 + \cos \Delta m(t_1 + t_2) + s^2[1 + 2 \cos \Delta m t_1 + 2 \cos \Delta m t_2 - \cos \Delta m(t_1 - t_2)]\}. \quad (A1)$$

$$P_{+}^{\pm\pm}(t_1, t_2) \propto (|A_l|^2)^2 \frac{e^{-\Gamma(t_1+t_2)}}{4(1+s^2)^2} \{1 - \cos \Delta m(t_1 + t_2) + s^2[3 - 2 \cos \Delta m t_1 - 2 \cos \Delta m t_2 + \cos \Delta m(t_1 + t_2)]\}. \quad (A2)$$

$$n_{+}^{-+} = \frac{1}{2(1+s^2)^2} \left[\frac{x_d^4 + x_d^2 + 2}{(1+x_d^2)^2} + s^2 \frac{4 + x_d^2}{1 + x_d^2} \right] \quad (A3)$$

$$n_{+}^{++} = n_{+}^{--} = \frac{1}{4(1+s^2)^2} \left[\frac{x_d^4 + 3x_d^2}{(1+x_d^2)^2} + s^2 \frac{3x_d^2}{1+x_d^2} \right] \quad (A4)$$

$$n_{+}^{-+}(\Delta t) = \frac{e^{-\Gamma|\Delta t|}}{2(1+s^2)^2} \left\{ \frac{2 + x_d^2}{1 + x_d^2} + s^2 \left[1 - \cos \Delta m \Delta t + \frac{2}{\sqrt{1+x_d^2/4}} (\sin(\Delta m|\Delta t| - \alpha)) + \cos(\Delta m|\Delta t| - \alpha) \right] \right\} \quad (A5)$$

$$n_{+}^{++}(\Delta t) = \frac{e^{-\Gamma|\Delta t|}}{4(1+s^2)^2} \left\{ \frac{x_d^2}{1+x_d^2} + s^2 \left[3 + \cos \Delta m \Delta t + \frac{2}{\sqrt{1+x_d^2/4}} (\sin(\Delta m|\Delta t| - \alpha)) + \cos(\Delta m|\Delta t| - \alpha) \right] \right\}, \quad (A6)$$

with $\alpha = \arctan \frac{x_d}{2}$.

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